File

SYDNEY TECHNICAL HIGH SCHOOL

HSC ASSESSMENT TASK 1

DECEMBER 2014

Mathematics

General Instuctions

- Working time - 70 minutes
- Write using black or blue pen
- Begin each question on a new page
- Board-approved calculators may be used
- All working should be shown in every question in section 2

Total marks - 60

Section 1

6 marks

- Attempt Questions 1 – 6
- Allow about 8 minutes for this section

Section 2

54 marks

- Attempt Questions 7 – 15
- Allow about 62 minutes for this section
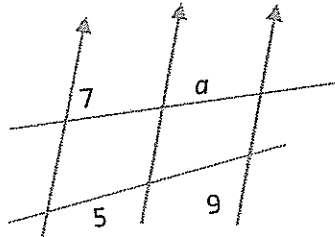
Name :_____

Teacher :_____

Section 1 – Multiple Choice

Indicate which of the answers A, B, C or D is the correct answer.

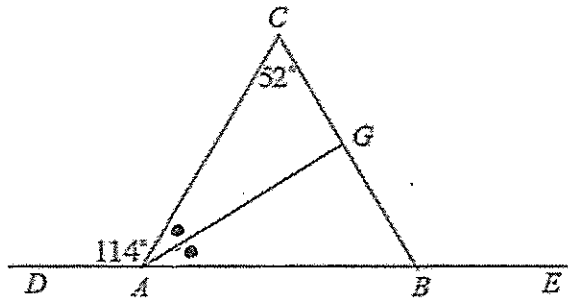Record this on your answer sheet

Question 1

Not to scale

The value of a in the above diagram is

    (A) 9
    (B) 11
    (C) 12
    (D) 12.6

Question 2

In the diagram, $\angle CAD = 114°$ & $\angle ACB = 52°$. DE is a straight line. AG bisects $\angle CAB$.

What is the value of $\angle AGB$?

(A) $33°$

(B) $52°$

(C) $62°$

(D) $85°$

Question 3

What are the co-ordinates of the focus of the parabola, $(x - 2)^2 = 8(2 - y)$?

(A) (2,4)
(B) (2,2)
(C) (2,0)
(D) (2,-2)

Question 4

What is the value of k if the sum of the roots of $x^2 - (k - 1)x + 2k = 0$ is equal to the product of the roots?

(A) -3
(B) -2
(C) -1
(D) 1

Question 5

The first term of a geometric series is 81 and the fourth term is $\frac{1}{9}$. What is the value of the common ratio?

(A) $\frac{1}{3}$
(B) $\frac{1}{9}$
(C) $\frac{1}{27}$
(D) $\frac{1}{81}$

Question 6

The centre and radius of the circle with equation $x^2 + y^2 - 6y = 4x$ are

(A) Centre (0,3) radius 2
(B) Centre (0, 9) radius 2
(C) Centre (2, 3) radius $\sqrt{13}$
(D) Centre (2, 3) radius 13

Section 2 :

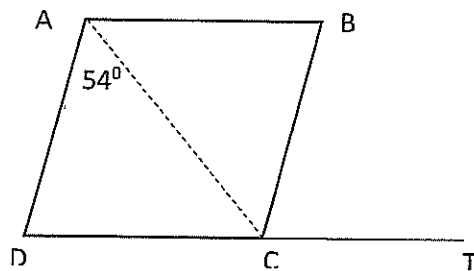**Question 7:** Start this question at the top of a new page

    a. Solve $6x^2 - 23x - 4 < 0$ 2

    b. Evaluate $\sum_{n=3}^{9} 3^{n-2}$ 2

    c. In the diagram ABCD is a rhombus where $\angle DAC = 54^0$ and DC is produced to T.

    Copy the diagram into your answer booklet

        i.   What is the value of $\angle DAB$?   1

        ii.   What is the value of $\angle BCT$? Give reasons.   1

Question 8: Start this question at the top of a new page

    The first three terms of an Arithmetic sequence are given as
    $x, 2x + 2, 4x - 1,$

    a. Find the value of x.   2

    b. Find the value of the 12th term in this sequence.   2

    c. Find the sum of the first 20 terms in this sequence.   2

Question 9: Start this question at the top of a new page

    a. If α and β the roots of the equation $4x^2 - 3x - 6 = 0$, find the value of

        i. $\alpha + \beta$ 1

        ii. $(\alpha - \beta)^2$ 3

    b. Write down the locus of a parabola with vertex (-1, 3) and directrix $y = 5$.   2

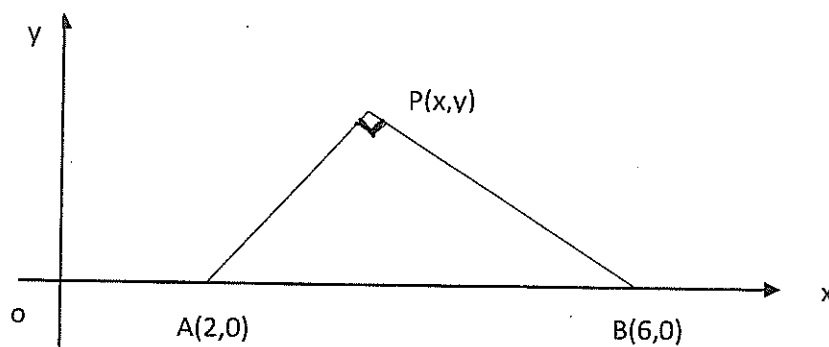**Question 10: Start this question at the top of a new page**

    a. Find the value of A, B and C given,

$$3x^2 - 4 \equiv A(x+1)^2 - B(x+2) + C \qquad\qquad\qquad 3$$

    b. Find the sum of this series

$$48 + 24 + 12 + \text{.........} + \frac{3}{32} \qquad\qquad\qquad 3$$

Question 11: Start this question at the top of a new page

  i.   Write down the gradient of PA in terms of x and y.   1
  ii.   Find the locus of all points P such that angle APB is a right angle.   2
  iii.   Deduce that the locus of all points P such that angle APB is a right angle is a circle.
          Write down the centre and radius of this circle.   3

Question 12: Start this question at the top of a new page
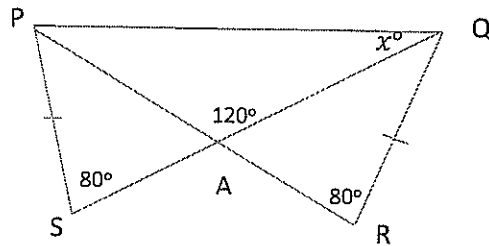
    a. A parabola P has equation $y = x^2 - 6x + 12$

       Write down

      i.   The co-ordinates of the vertex of P.   2
      ii.   The co-ordinates of the focus of P.   1
      iii.   The equation of the directrix of P.   1

    b. Find the equation of the tangent to the parabola $x^2 = -8(y - 4)$ at the point (4, 2).   2

Question 13: Start this question at the top of a new page

NOT TO SCALE

PR and QS are straight lines intersecting at a point A. Also PS = QR,

$\angle PSA = \angle QRA = 80^0, \angle PAQ = 120^0$ and $\angle PQA = x$.
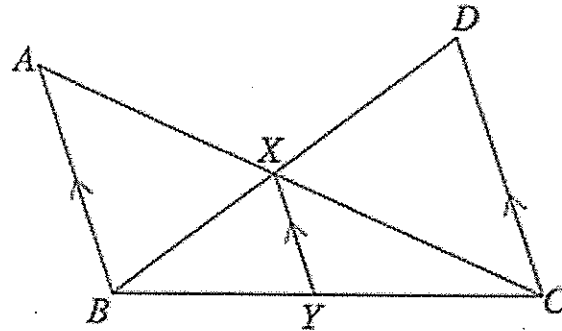
a. Copy the diagram into your answer page

b. Prove that $\triangle PSA$ is congruent to $\triangle QRA$ 3

c. Hence, prove that $\triangle PAQ$ is isosceles and find the value of x.   3

Question 14: Start this question at the top of a new page

a. A seedling with height 46 mm is planted and its growth tracked. Its height after one week was 59.8 mm and after a further week its height was recorded as 77.74 mm. If the seedling continues to grow at the same rate, find,

     i. Its height after 6 weeks   1

     ii. The week in which it's height is first recorded as over 1 metre   2

b. Find the value/s of m required for the line $y = mx - 12$ to be a tangent to the parabola $y = 2x^2 - x - 10$ 3

Question 15: Start this question at the top of a new page

In the diagram below $AB \mathbin{//} XY \mathbin{//} DC$.

NOT TO SCALE

Copy the diagram into your answer page

a. Prove that $\triangle AXB$ is similar to $\triangle CXD$ 2

b. If $XB = 12$ cm, $XC = 30$ cm, $BY = 8$ cm and $YC = 24$ cm,
    find the length of AX and DX, giving reasons.   3

c. Hence, find $AB : DC$ 1

END OF ASSESSMENT TASK

Section 1

Question 1   D
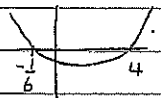2   D
3   C
4   C
5   B
6   C.

Section 2.

Question 7

a) $6x^2 - 23x - 4 < 0$

$(6x+1)(x-4) < 0$

$-\frac{1}{6} < x < 4$

~~Question 8~~

b) $\sum_{3}^{9} 3^{n-2} = 3 + 3^2 + 3^3 + 3^4 + 3^5 + 3^6 + 3^7$

GP

$\frac{a(r^n - 1)}{r-1} = \frac{3(3^7 - 1)}{2}$

$= 3279$ (or add using your calculator.

c) 1. $\angle DAB = 2 \angle DAC$

$= 108°$ (diagonal of a rhombus bisects interior angles).

(Needs reason).

ll. $\angle BCD = \angle DAB$ (opposite angles in a rhombus

$= 108°$ equal)

$\angle BCT + \angle BCD = 180°$ (straight line)

$\therefore \angle BCT = 72°$.

Question 8

a. AP $T_2 - T_1 = T_3 - T_2$

$(2x+2) - x = (4x-1) - (2x+2)$

$x + 2 = 2x - 3$

$x = 5$

b. $5, 12, 19, \ldots$

$T_n = a + (n-1)d$

$= 5 + 11 \times 7$.

$= 82$

c. $S_n = \frac{n}{2}(2a + (n-1)d)$

$= \frac{20}{2}(10 + 19 \times 7)$

$= 1430$

Question 9

a. $4x^2 - 3x - 6 = 0$

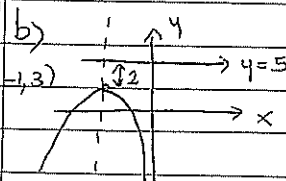1. $\alpha + \beta = -b/a$

$= 3/4$

ll. $(\alpha - \beta)^2 = \alpha^2 - 2\alpha\beta + \beta^2$

$= \alpha^2 + \beta^2 - 2\alpha\beta$

$= (\alpha + \beta)^2 - 4\alpha\beta$ $\alpha\beta = c/a$

$= \left(\frac{3}{4}\right)^2 - 4\left(\frac{-3}{2}\right)$ $= -3/2$

$= \frac{105}{16}$

b)

$(-1,3)$ $y = 5$

$(x+1)^2 = -8(y-3)$

One here.

Question 10

a. $3x^2 - 4 \equiv A(x+1)^2 - B(x+2) + C$

$RHS = A(x^2 + 2x + 1) - Bx - 2B + C$

Equating coefficients

$3 = A$ $\qquad$ $\therefore A = 3$ ✓

$0 = 2A - B$ $\qquad$ $\therefore B = 6$ ✓

$-4 = A - 2B + C$

$-4 = 3 - 2(6) + C$

$C = 5$ $\qquad$ $\therefore C = 5$ ✓

b. $48 + 24 + 12 + \ldots + \frac{3}{32}$.

GP $\quad a = 48 \quad r = \frac{1}{2} \quad T_n = \frac{3}{32}$ ✓

$ar^{n-1} = \frac{3}{32}$

$48 \left(\frac{1}{2}\right)^{n-1} = \frac{3}{32}$

$\left(\frac{1}{2}\right)^{n-1} = \frac{1}{512}$

$2^{n-1} = 2^9$

$n = 10$ ✓

$\therefore S_n = \dfrac{a(r^n - 1)}{r - 1}$ $\quad$ OR $\quad \dfrac{a(1 - r^n)}{1 - r}$

$= \dfrac{48\left(\frac{1}{2}^{10} - 1\right)}{\frac{1}{2} - 1}$ ✓ $\qquad \dfrac{48\left(1 - \frac{1}{2}^{10}\right)}{1 - \frac{1}{2}}$

$= \dfrac{3069}{32}$.

Question 11.

a. $m_{PA} = \dfrac{y - 0}{x - 2}$

$= \dfrac{y}{x - 2}$

$= \dfrac{y}{x - 2}$ ✓

b. $M_{PA} \times M_{PB} = -1$ ✓

$\dfrac{y}{x-2} \times \dfrac{y}{x-6} = -1$

$y^2 = -1(x^2 - 8x + 12)$

$y^2 = -x^2 + 8x - 12$

$x^2 - 8x + y^2 = -12$ ← ✓

$\quad + 16 \qquad\qquad + 16$

c. $(x - 4)^2 + y^2 = 4$ ✓ $\qquad C = (4, 0)$ ✓

$\qquad\qquad\qquad\qquad\qquad R = 2$ ✓

↗ this maybe done in part (b)

Needs to be in this format.

Question 12

a. $y = x^2 - 6x + 12$

$x^2 - 6x + 9 = y - 12 + 9$

$(x - 3)^2 = 1(y - 3)$ $\qquad a = \frac{1}{4}$

i. vertex $(3, 3)$

ii. focus $\left(3, 3\frac{1}{4}\right)$

iii. $y = 2\frac{3}{4}$

b. $x^2 = 8(4 - y) \longrightarrow x^2 = 4 - y$

$x^2 = -8(y - 4)$ $\qquad\qquad \dfrac{x^2}{8}$

$\qquad\qquad\qquad\qquad y = 4 - \dfrac{x^2}{8}$ ✓

$\therefore y - y_1 = m(x - x_1)$ ⎫

$y - 2 = -1(x - 4)$ ⎬ any $\qquad \dfrac{dy}{dx} = -2x/8$ at $x = 4$

$y - 2 = -x + 4$ ⎭ $\qquad\qquad m_T = -1$

$x + y - 6 = 0$ ✓

Question 13.

a. 1.
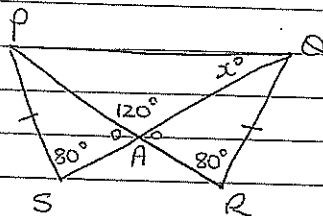
(No mark for diagram)

ii. In $\triangle PSA$ and $\triangle QRA$

$PS = QR$ given

$\angle PSA = \angle QRA$ (given $80°$) — both ✓

$\angle PAS = \angle QAR$ (vertically opposite) ✓

$\therefore \triangle PSA \equiv \triangle QRA$ (AAS). ✓

iii. $PA = QA$ ✓ corresponding sides of congruent triangles ✓

$\therefore \triangle PAQ$ is isosceles, 2 sides equal.

$\angle APQ = \angle AQP$ (equal angles opposite equal sides)

$120 + 2x = 180$

$2x = 60$

$x = 30$ ✓

Question 14

a.

46mm	59.8mm	77.74mm	
planted	Week1	Week 2.	GP
Term 1	T_2	T_3	$a = 46$
			$r = 1.3$.

1. Week 6 → Term 7 ar^{n-1}

$= 46(1.3)^6$

$= 222.033214$ mm.

ii. Week W → Term $(w+1)$

$46(1.3)^{w+1-1} > 1000$

$1.3^w > 21.73913\ldots$

$1.3^{12} = 23.298$

$1.3^{12} > 21.73913$

ie   12 weeks needed.

b) $2x^2 - x - 10 = mx - 12$

$2x^2 - x - mx + 2 = 0$

$2x^2 - (1+m)x + 2 = 0$

for tangent $\Delta = 0$

$b^2 - 4ac = 0$

$(1+m)^2 - 4(2)(2) = 0$
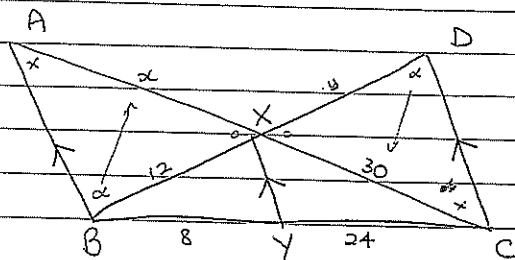
$1 + 2m + m^2 - 16 = 0$

$m^2 + 2m - 15 = 0$

$(m+5)(m-3) = 0$
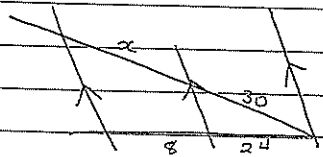
$m = -5, m = 3$

Question 15

I. In $\triangle AXB$ and $\triangle CXD$

   $\angle AXB = \angle CXD$ (vertically opposite)

   $\angle BAX = \angle DCX$ (alternate angles $AB \parallel CD$).

   $\therefore \triangle AXB \parallel\parallel\parallel \triangle CXD$ (equiangular).

II.

$$\frac{x}{30} = \frac{8}{24}$$

ratio of intercepts on parallel lines

$$24x = 240$$

$$x = 10$$

$$AX = 10\,cm$$

Likewise

$$\frac{y}{12} = \frac{24}{8}$$

$$8y = 288$$

$$y = 36 \qquad XD = 36\,cm$$

III. Ratio $AB : CD = AX : XC$ ratio of corresponding sides in similar triangles.

$$= 1 : 3.$$